3.8 Derivatives of Inverse Functions

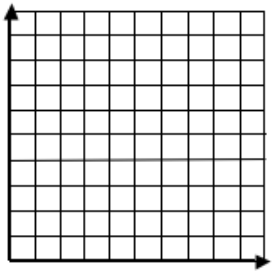
Objectives:

- I can find the derivative of an inverse function

- I can find the derivative of an inverse trig function

Part I:

a. Graph $f(x) = x^2 \; for \; x \geq 0$



b. What is f(2)?

c. Draw the point on the graph that you just found. Label it A.

d. What is f '(2)?

e. Draw the tangent line at x=2 (point a)

f. Find $f^{-1}(x)$

g. Graph $f^{-1}(x)$

h. Draw the point on $f^{-1}(x)$ that is the reflection of point A. Label it point B.

i. Find the slope of the tangent line of $f^{-1}(x)$ at point B.

j. Draw the line.

k. What is the relationship between the slopes of the tangent lines of this "reflected pair" A and B?

l. Do you think that relationship is true of all "reflected pairs" of points?

Given $f(x) = x^4 - 3x^2 + 4x + 2$ find the following:

a) f(1), f '(1)

b) $f^{-1}(4), \left(f^{-1}\right)'(4)$

Summary:

$$f(a) = b \longrightarrow f^{-1}(b) = a$$

$$f'(a) = c \longrightarrow \left(f^{-1}\right)'(b) = \frac{1}{c}$$

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Given $f(x)=\cos x+x$ find the following:

a) $f(0),\ f'(0)$

b) $f^{-1}(1),\left(f^{-1}\right)'(1)$

Part II: Inverse trig functions

Find $\frac{dy}{dx}$ *if* $y=\sin^{-1}x$

Find $\frac{dy}{dx}$ *if* $y=\tan^{-1}x$

Rule Sheet: 67-72

67. $\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$

68. $\frac{d}{dx}(\cos^{-1}x)=\frac{-1}{\sqrt{1-x^2}}$

69. $\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$

70. $\frac{d}{dx}(\cot^{-1}x)=\frac{-1}{1+x^2}$

71. $\frac{d}{dx}(\sec^{-1}x)=\frac{1}{|x|\sqrt{x^2-1}}$

72. $\frac{d}{dx}(\csc^{-1}x)=\frac{-1}{|x|\sqrt{x^2-1}}$

Helpful identities

$\sec^{-1}x=\cos^{-1}\left(\frac{1}{x}\right)$

$\csc^{-1}x=\sin^{-1}\left(\frac{1}{x}\right)$

$\cot^{-1}x=\tan^{-1}\left(\frac{1}{x}\right)$

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Derive:

$f(x) = \cos^{-1}(3x)$

$y = \cos^{-1}\left(\frac{3}{x}\right)$

$g(x) = \sin^{-1} x^2$

$g(x) = \cos^{-1}\left(\frac{4}{x^2}\right)$

Derive:

$f(x) = \csc^{-1}(3x+2)$

$y = \tan^{-1}\sqrt{x^2+2}$

$y = \sec^{-1}(3x^2)$

$f(x) = \sin^{-1}\left(\frac{1}{x}\right)$