4.4 Optimization

Objectives:

- I can write a model to represent a situation

- I can solve a model to maximize or minimize a value

Consider an open box made by cutting congruent squares out of the corners of an 8 1/2 x 11 in sheet of paper.

What size square could be cut out to maximize the volume?

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Process for optimizing

1- Understand the problem- Draw a diagram and assign variables.

2- Write a model (function) to represent the problem

-Start with the equation of what you are trying to maximize

- Use substitution to get the equation as a function of only one variable.

3- Identify critical points and closed endpoints as candidates.

4- Test critical points to determine max/min

- Plug into function or

- First derivative test or

- Second derivative test

5- Interpret/state solution

1. You are designing an open box with a square base and a required volume of 108 cubic inches. What dimensions would minimize the materials needed?

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2. A rectangle is inscribed between $y = -x^2 + 9$ and the x-axis. Find the maximum area of the rectangle.

3. A rectangle is inscribed between y = sinx and the x-axis over $[0, \pi]$. Find the maximum area of the rectangle.

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4. Design a 2-liter can with minimum surface area.

5. What are the dimensions of the lightest open top right cylindrical can that will hold 2197 cubic cm?

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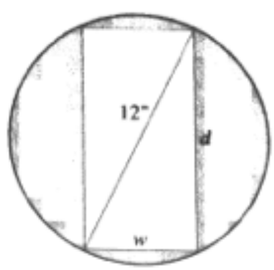
6. Suppose $r(x)=\dfrac{x^2}{x^2+1}$ represents revenue and $c(x)=\dfrac{(x-1)^2}{3}-\dfrac{1}{3}$ represents cost, with x measured in thousands of units. What is the production level that maximizes profit?

7. Bobby is 3 miles off shore in a boat and wants to reach a campsite that is 5 miles down a straight shoreline from the point nearest to his boat. He can row 3 mph and jog 4 mph. How far from camp should he land his boat to minimize the time to reach the camp?

8. How close does the semicircle $y=\sqrt{16-x^2}$ come to the point $(1,\sqrt{3})$



38. ***Stiffness of a Beam*** The stiffness S of a rectangular beam is proportional to its width times the cube of its depth.

(a) Find the dimensions of the stiffest beam that can be cut from a 12-in. diameter cylindrical log.

(b) **Writing to Learn** Graph S as a function of the beam's width w, assuming the proportionality constant to be $k = 1$. Reconcile what you see with your answer in part (a).