8.2 L'Hopital's Rule

- I can recognize limits in indeterminate form
- I can evaluate limits applying L'Hopital's rule

Evaluate the limit using substitution:

$$\lim_{x \to 0} \frac{\sin x}{x}$$

$$\lim_{x \to \infty} \frac{\ln x}{2\sqrt{x}}$$

Evaluate the following limits

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4}$$

THEOREM 1 (l'Hopital's Rule for zero over zero): Suppose that $\lim_{x \to a} f(x) = 0$, $\lim_{x \to a} g(x) = 0$, and that functions f and g are differentiable on an open interval I containing a. Assume also that $g'(x) \neq 0$ in I if $x \neq a$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

so long as the limit is finite, $+\infty$, or $-\infty$. Similar results hold for $x \to \infty$ and $x \to -\infty$.

THEOREM 2 (l'Hopital's Rule for infinity over infinity): Assume that functions f and g are differentiable for all x larger than some fixed number. If $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = \infty$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

so long as the limit is finite, $+\infty$, or $-\infty$. Similar results hold for $x \to \infty$ and $x \to -\infty$.

In both forms of l'Hopital's Rule it should be noted that you are required to differentiate (separately) the numerator and denominator of the ratio if either of the indeterminate forms $\frac{"0"}{0}$ or $\frac{"\infty"}{\infty}$ arises in the computation of a limit. Do not confuse l'Hopital's Rule with the Quotient Rule for derivatives. Here is a simple illustration of Theorem 1.

Use L'Hopital's rule evaluate the following and support graphically

$$\lim_{x \to 0} \frac{\sin x}{x} \qquad \lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4}$$

$$\lim_{x \to \infty} \frac{\ln x}{2\sqrt{x}}$$

Apply L'Hopital's rule to evaluate the following limits

1. $\lim_{x \to 1} \frac{\ln x}{x - 1}$

2. $\lim_{x \to 0} \frac{\sqrt{1 + x} - 1}{x}$

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Apply L'Hopital's rule to evaluate the following limits

3. $\lim_{x \to \pi} \frac{\pi - x}{\sin x}$

Apply L'Hopital's rule to evaluate the following limits

6. $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$